An Adapted Model for Simulation of the Interaction between a Wall and the Building Heating System

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ABSTRACT

A well-designed building, particularly the thermal aspect, requires a correct definition of the walls, a fitted heating system, and an adapted control system. Nevertheless, each problem is closely related to the others and must not be solved separately. It is also very important to be able to assess the impact of the interaction of the different chosen solutions. In this objective, a rapid and accurate simulation tool has been developed, taking into account the interaction between the envelope of a building and a water heating system. The work presented in this paper is more precisely related to the choice of the dynamic modeling of conduction through the walls. The chosen model must be adapted to the thermal driving forces to which the walls are subjected:

-- indoor high-frequency excitations (e.q., regulation)
-- outdoor low-frequency excitations (e.q., air temperature).

We suggest one model with three thermal capacities and two thermal resistances to describe conduction through the walls. An identification process using the results of a reference model (finite differences) allows us to obtain these five parameters. The quality of the model is then discussed through some examples using theoretical inputs (temperature step or ideal temperature swings) or real weather data. The model appears to be well adapted to driving forces due to the control loop. It gives good results for the simulation of the thermal behavior of a building and its heating system.

PRESENTATION OF THE BUILDING MODEL

The choice of a water heating system needs a study that is often carried out at steady-state conditions with minimal external temperature. Nevertheless, if we want to consider the dynamics of the heating system in conjunction with the dynamics of the building envelope, it is necessary to use numerical tools. The model we have designed allows such a study and includes some points of view often neglected: heat regulation, hydraulic network of a central heating system, etc.

Like the real building, the building model that we suggest can take into account several "thermal zones." A thermal zone is a part of the building (one room or several rooms taken together) in which the inside temperature is practically homogeneous. Moreover, the heating of that part of the building can be regulated independently of the other parts. This enables us to study complex heating systems, such as central heating, the influence of the ventilation

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between the different zones, and the heating intermittence in each zone.

The design of the building model is based on several simplifying assumptions which are in keeping with the chosen degree of precision and the present level of knowledge. In particular, the air temperature and the radiant temperature are considered to be uniform in each zone. The radiant temperature is considered to follow the equation:

$$T_{RM}(t) = \frac{\sum_{p=1}^{n} S_p \, T_{Sp}(t)}{\sum_{p=1}^{n} S_p} \qquad (1)$$

where T_{Sp} = the inner surface temperature of the p^{th} wall

S_p = the surface area of the p^{th} wall

n = the number of walls that delimit the zone.

The building model is constructed by assembling a model of each wall of the real building.

THE PROPOSED WALL MODEL

Definition of the Wall Model

Following the work of Laret(1980), Roux(1984) showed that a wall model with one thermal resistance and two thermal capacitances used with one-hour steps may predict the evolution of the heating needs of a building. We call this sort of model an **R2C model**. The resistance is equal to the real thermal resistance of the wall. The two capacitances are located on the surfaces of the wall. Their calculations are clearly explained in Roux(1984).

To study the coupling of the heating system with the building, the use of short timesteps (a few minutes) is necessary. Since the fluctuations of the inside temperature must be precisely calculated, the dynamic behavior of the envelope model must be improved. As a result, we have developed a model (**2R3C model**) with two thermal resistances (R_A, R_B) and three thermal capacitances (C_A, C_M, C_B) (Figure 1). Two of the capacitances (C_A, C_B) are located on the surface of the wall. The plane that contains the C_M capacity divides the wall into:

-- The A part, E_A thick, which is subjected to the environmental excitations and has N_A layers of different materials (concrete, insulator)

-- The B part, E_B thick, which is subjected to the inner excitations and has N_B layers of different materials (insulator, plaster).

To properly calculate the heating consumption, it is necessary to correctly evaluate the heat flux that goes through the wall. Also, the thermal resistances must follow the equation:

$$R_A + R_B = \sum_{p=1}^{N_A+N_B} \frac{e_p}{l_p} \qquad (2)$$

where e_p = the thickness of the p^{th} layer
l_p = the thermal conductivity of the p^{th} layer.

To have fewer unknowns, we impose:

$$R_A = \sum_{p=1}^{N_A} \frac{e_p}{l_p} \qquad (3)$$

$$R_B = \sum_{p=N_A+1}^{N_A+N_B} \frac{e_p}{l_p} \qquad (4)$$

Using this model, it is easy to express the temperature in terms of the temperatures of the capacities ($T_A(t)$, $T_M(t)$, $T_B(t)$). In particular, the temperature of the plane that separates two layers (Figure 2) is equal to:

-- For $1 \le j \le N_A$

$$T_{sj}(t) = T_A(t) + \frac{\sum_{k=1}^{j} \frac{e_k}{l_k}}{R_A} (T_M(t) - T_A(t)) \qquad (5)$$

-- For $N_A+1 \le j \le N_A + N_B$

$$T_{sj}(t) = T_M(t) + \frac{\sum_{k=N_A+1}^{j} \frac{e_k}{l_k}}{R_B} (T_B(t) - T_M(t)) \qquad (6)$$

The chosen model must simulate precisely the capacity of the wall to store or to release the heat. This property is explained by the values of the capacities and their temperatures.

The study of the variation between two times (t_1,t_2) of the heat stored in the wall allows us to express these capacities in terms of the thermal characteristics of each layer.

Indeed,the variation of the heat stored in a **j** layer of the **A** part is:

$$\delta q_j = o_j\ C_j\ e_j\ [\frac{(T_{sj-1}(t_2) - T_{sj-1}(t_1)) + (T_{sj}(t_2) - T_{si}(t_1))}{2}]$$

$$= o_j\ C_j\ e_j\ [\frac{\delta T_{sj-1}(t_1,t_2) + \delta T_{sj}(t_1,t_2)}{2}] \qquad (7)$$

where C_j = the thermal capacity of the j^{th} layer
o_j = the density of the j^{th} layer.

From the distribution of the temperature defined by (Equation 5):

$$\frac{\delta T_{sj-1}(t_1,t_2) + \delta T_j(t_1,t_2)}{2} =$$

$$= \delta T_A(t_1,t_2) + \frac{\frac{e_j}{2\ l_j} + \sum_{k=1}^{j-1} \frac{e_k}{l_k}}{R_A} (\ \delta T_M(t_1,t_2) - \delta T_A(t_1,t_2)\ )$$

From which one concludes that:

$$\delta q_{1j} = o_j\ C_j\ e_j\ [(1 - \beta_j)\ \delta T_A(t_1,t_2) + \beta_j\ \delta T_M(t_1,t_2)\ ) \tag{8}$$

where

$$\beta_j = \left(\frac{e_j}{2\ l_j} + \sum_{k=1}^{j-1} \frac{e_k}{l_k} \right)/R_A \tag{9}$$

In the same way, the variation of the heat stored in a j layer of the B part is:

$$\delta q_{2j} = o_j\ C_j\ e_j\ [\ (1 - \Gamma_j)\ \delta T_M(t_1,t_2) + \Gamma_j\ \delta T_B(t_1,t_2)\ ] \tag{10}$$

where

$$\Gamma_j = \left(\frac{e_j}{2\ l_j} + \sum_{k=N_A+1}^{j-1} \frac{e_k}{l_k} \right)/R_B \tag{11}$$

So, the entire variation of the energy stored in the wall between t_1 and t_2 moments is:

$$\delta Q_w = \sum_{j=1}^{N_A} \delta q_{1j} + \sum_{j=N_A+1}^{N_A+N_B} \delta q_{2j}$$

$$= \left[\sum_{j=1}^{N_A} o_j\ C_j\ e_j (1-\beta_j)\ \right] \delta T_A(t_1,t_2)$$

$$+ \left[\sum_{j=1}^{N_A} o_j\ C_j\ e_j\ \beta_j + \sum_{j=N_A+1}^{N_A+N_B} o_j\ C_j\ e_j\ (1 - \Gamma_j)\ \right] \delta T_M(t_1,t_2)$$

$$+ \left[\sum_{j=N_A+1}^{N_A+N_B} o_j\ C_j\ e_j\ \Gamma_j\ \right] \delta T_B(t_1,t_2) \tag{12}$$

According to the model, this variation can also be expressed by:

$$Q_w = C_A\ \delta T_A(t_1,t_2) + C_M\ \delta T_M(t_1,t_2) + C_B\ \delta T_B\ (t_1,t_2) \tag{13}$$

By identification of the parameters, we obtain:

$$C_A = \sum_{j=1}^{N_A} o_j\ C_j\ e_j\ (1 - \beta_j) \tag{14}$$

$$C_M = \sum_{j=1}^{N_A} o_j\ C_j\ e_j\ \beta_j + \sum_{j=N_A+1}^{N_A+N_B} o_j\ C_j\ e_j\ (1 - \delta_j) \tag{15}$$

$$C_B = \sum_{j=N_A+1}^{N_A+N_B} o_j\ C_j\ e_j\ \delta_j \tag{16}$$

The sole unknown of the model is the position of the plane that contains the C_M capacity. We can locate it with the help of the α coefficient, as follows:

$$E_A = \alpha\ E = \sum_{j=1}^{N_A} e_j \tag{17}$$

$$E_B = (1 - \alpha)\ E = \sum_{j=N_A+1}^{N_A+N_B} e_j \qquad (18)$$

and: $E = E_A + E_B$ (total thickness of the wall) (19)

This position has an important effect upon the dynamic of the model and consequently upon its quality.

Identification of the Parameters of the Wall Model

We carry out a systematic study of the quality of the model, according to the position of C_M capacity. The identification of the optimum position takes into account the demands made upon the wall and the necessary output. We know that there is an optimum model for a given "driving force-output" couple. For another couple, the optimum model is different.

In the study of the building-system coupling, the evaluation of the inner air temperature and the inner surface temperature of the walls is given priority. So, the required output is the surface temperature on the inner side of the wall.

Furthermore, we know that such a wall model is reliable when calculating the flux and the inner surface temperature of the walls, resulting from real outside excitations. In that case, the periodic thermal signals that go through the walls are considerably reduced for periods about an hour or less.

In accordance with our aim, giving priority to the inner excitations of the building seems to be judicious. In the building model, each wall surface is subjected to Fourier boundary conditions (ambient temperatures). To identify the parameters, we have chosen the following ambient temperatures:

-- on the outer side : $T_E(t) = 0°C$, $\quad t$

-- on the inner side : $T_I(t) = 0°C$ when $t \leq 0$
$T_I(t) = 1°C$ when $t > 0$

Thus, the evolution of the surface temperature $T_{SI}(t,\alpha)$ on the inside varies according to C_M capacity position. These variations are compared to the reference evolution $T_{réf}$, which has been calculated from a precise model (finite differences). This comparison consists of calculating the I integral, which is our identification test:

$$I(\alpha) = \int_0^{+\infty} \left| T_{ref}(t) - T_{SI}(t,\alpha) \right|^2 dt \qquad (20)$$

The optimum model minimizes the I integral. The optimum parameter for such a model is $\alpha = \alpha_{op}$. The $I(\alpha)$ function possesses only one minimum. Furthermore, there is an interval $[\alpha m, \alpha M]$ that contains α_{op} and where the model remains accurate (Figure 3). It is easy, therefore, to define α_{op} and consequently to identify a quasi-optimum model.

Presentation of Results for Different Inertia Walls

Studied Walls. The inertia of the walls is a factor of great consideration both for the comfort and the heating regulation. Generaly speaking, the model must be adapted at all sorts of walls whatever their inertia.

In order to test the quality of our model, we have studied two walls of differing inertia (Figure 4). For each test, we have compared the response of our model with those provided by a precise model (finite differences) and the R2C model.

Evolution of the Inner Surface Temperature When the Inner Ambient Temperature is a Step._The first test shows the response of the model to one step in the inner ambient temperature. The outer ambient temperature remains at zero. This test enabled us to identify the model (Figure 5). We observe the excellent response of our model compared with that of the precise model and an improvement compared to the R2C model whatever the wall inertia.

Evolution of the Inner Surface Temperature When the Inner Ambient Temperature is an Ideal Temperature Swing (P Period). The second test studies the model response to a sinusoidal inner ambient temperature in function of its period:

$$T_I(t) = Tc \sin\left(2 \pi \frac{t}{P} \right) \tag{21}$$

This analysis shows the particular quality of the model to simulate the periodic variation generated by the heating regulation (Figure 6). For the weak inertia wall, the model response is perfect when the period is greater than one hour, and accurate when it is between 10 minutes and one hour. For the strong inertia wall, the response model is accurate when the period is greater than one hour. But when the period is less than one hour, the model quality declines and rapidly becomes mediocre. This study corroborates the marked improvement compared to the R2C model.

Evolution of the Inner Surface Temperature When The Inner and Outer Ambient Temperatures are Complex. In order to illustrate the qualities and defaults of the model, we have subjected each wall to complex excitations (Figure 7). The inner ambient temperature is the sum of constant and different ideal swings; it follows the equation:

$$T_I(t) = T_C + \sin(2\pi\frac{t}{P_1}) + \sin(2\pi\frac{t}{P_2}) + \sin(2\pi\frac{t}{P_3}) + \sin(2\pi\frac{t}{P_4}) \tag{22}$$

where P_1 = 15 min, P_2 = 1 h, P_3 = 6 h, P_4 = 24 h, T_C = 20°C.

The outer ambient temperature is real weather data.

This third test is rich in lessons and allows us to conclude (Figure 8). Indeed, for the weak inertia wall, the model response is perfect. For the strong inertia wall, the model simulates accurately the variations when their periods are greater than or equal to one hour. But the model can not simulate the rapid variation of 15 minutes.

At the origin of time, the wall is at 18°C. This initial state causes an inner and an outer temperature step. This allows us to establish the accurate response of the model to brusque outer conditions.

Furthermore, for the strong inertia wall, the test shows clearly that the R2C model is unable to simulate the inner surface temperature.

CONCLUSION

We establish that the quality of our proposed model is a function of the inertia of the presented wall. Despite the considerable simplification of the wall model, the simulation of the inner surface temperature is excellent for the weak inertia walls and accurate enough for the strong inertia walls. In each case, there is a marked improvement compared to the R2C model. This improvement is indispensable to simulate the inner surface temperature of the walls with a few minute timestep.

The dynamics of our model seems to be sufficient to study the interaction between the building, its heating system, and the regulation system.

The reduced size of the model makes it possible to describe a building made up of several zones, and it can be used on a micro-computer with reasonable running times.

Thus, we have connected the building to a water heating system. We can study the regulation system, the regulation strategy, compare the different components of the heating system,and so on.

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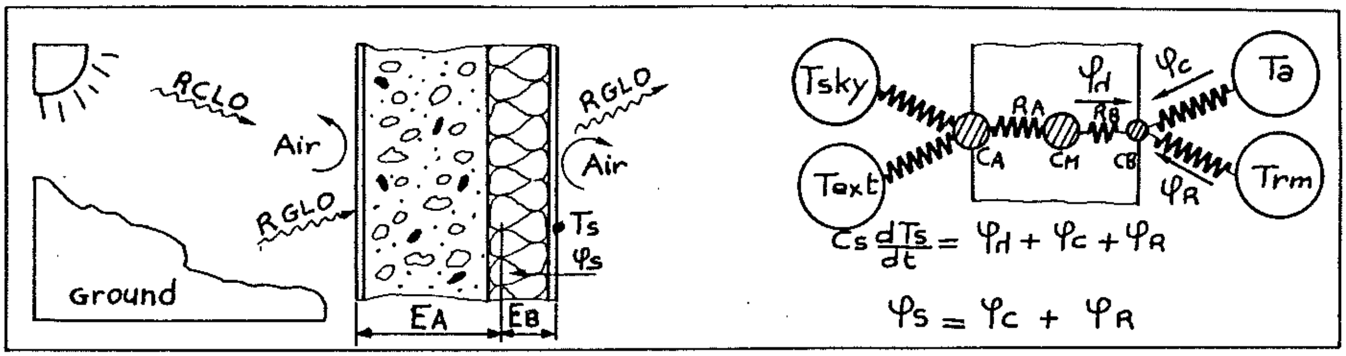


Figure 1. Analog diagram of the wall model

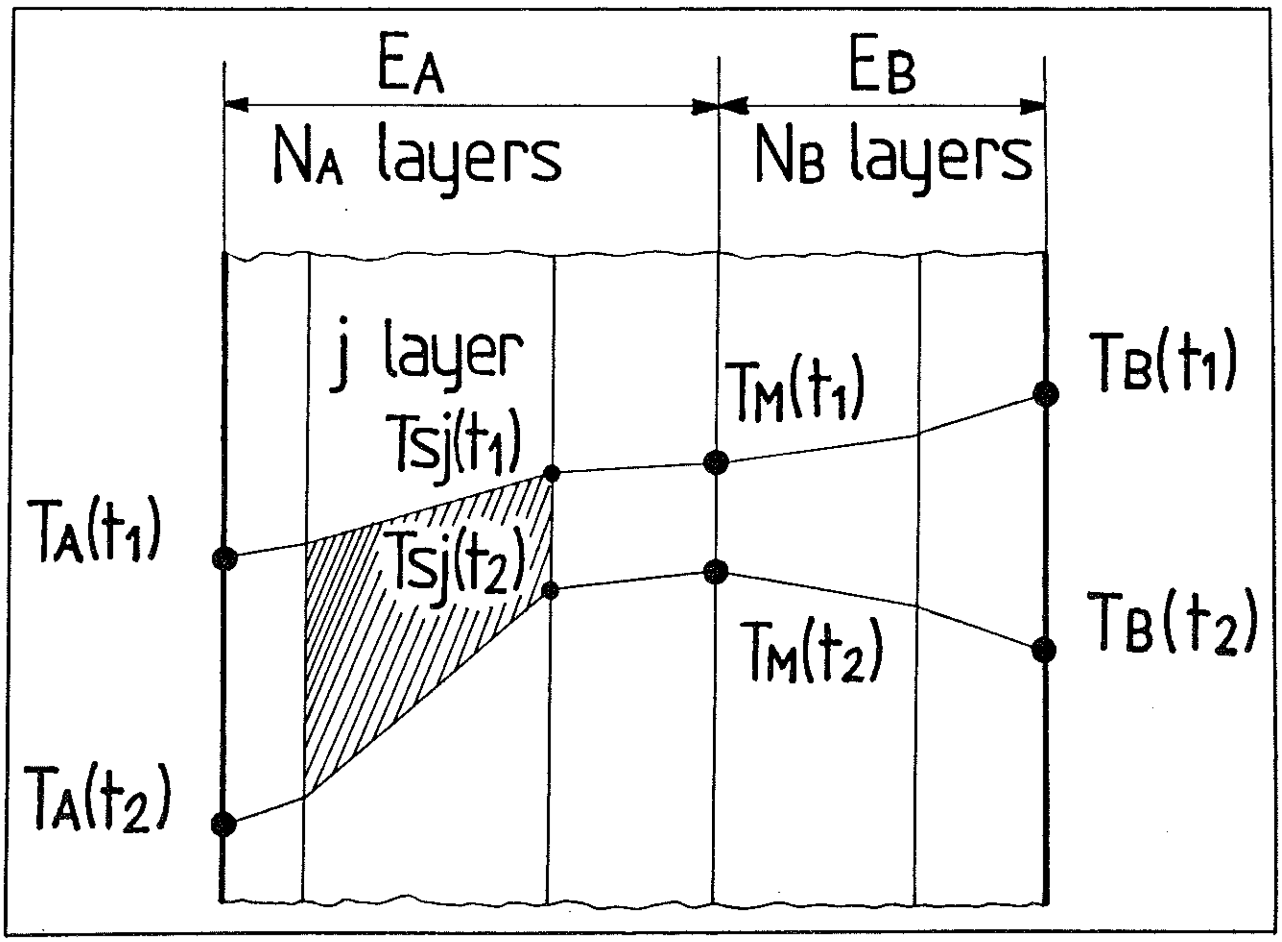


Figure 2. Temperature in the wall

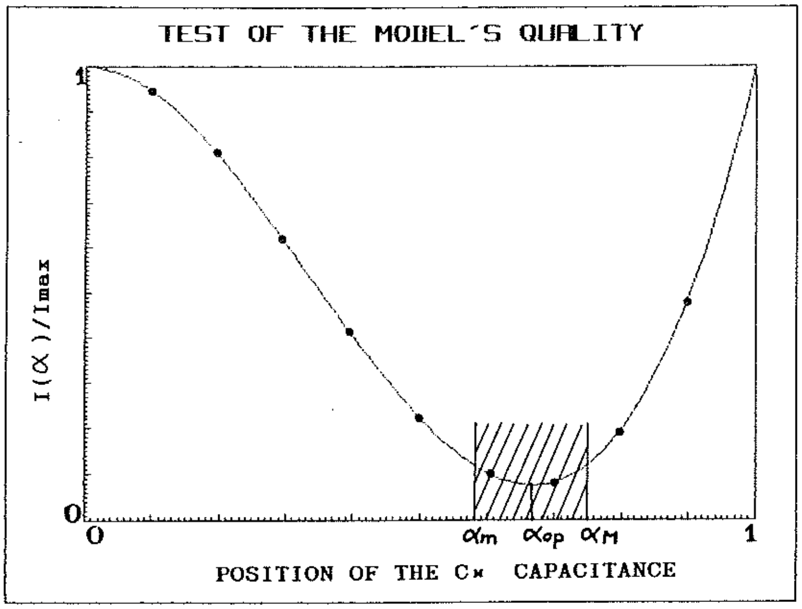
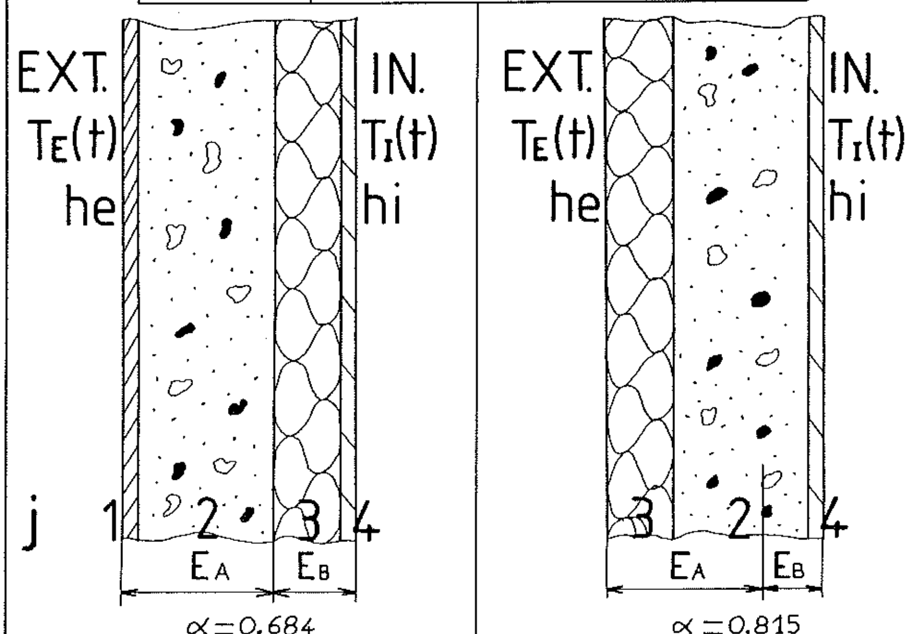


Figure 3. Variation of the model quality

j indice	1	2	3	4
e (m)	0.015	0.18	0.08	0.01
λ (W/m°C)	1.15	1.75	0.042	0.35
ρ (Kg/m³)	1900	2500	15	900
C (J/Kg°C)	920	920	920	920



Weak inertia wall

R_A = 0.1159 m^2°C/W
R_B = 1.9333 m^2°C/W
C_A = 2.0845 E+05 J/°Cm²
C_M = 2.3239 E+05 J/°Cm²
C_B = 8.7627 E+03 J/°Cm²

Strong inertia wall

R_A = 1.9848 m^2°C/W
R_B = 0.0514 m^2°C/W
C_A = 7.0637 E+03 J/°Cm²
C_M = 3.8990 E+05 J/°Cm²
C_B = 2.6424 E+04 J/°Cm²

Figure 4. The characteristics of the studied wall

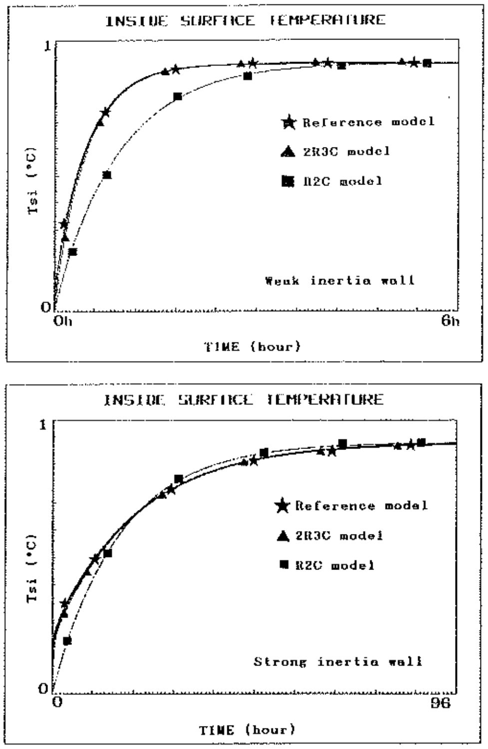


Figure 5. Variation of the inner surface temperature

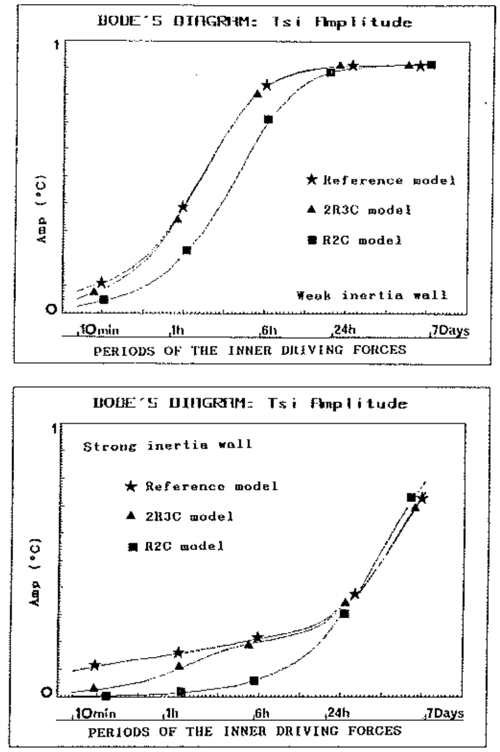


Figure 6. Model quality in function of the excitation period

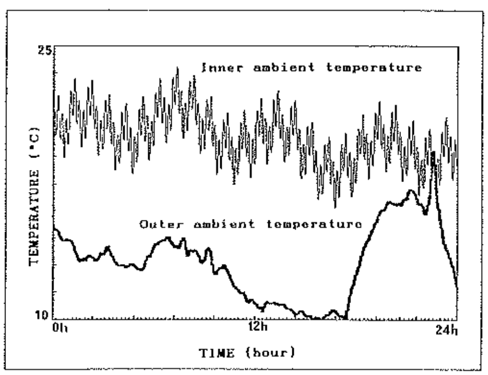


Figure 7. Inner and outer ambient temperature

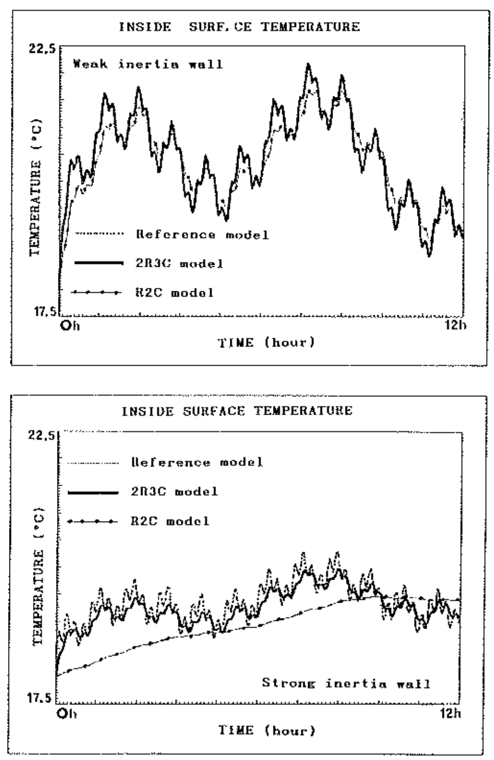


Figure 8. Model response to complex excitations